Theory for Photon-Assisted Macroscopic Quantum Tunneling in a Stack of Intrinsic Josephson Junctions

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We propose a theory for photon-assisted macroscopic quantum tunneling (MQT) in a stack of capacitively-coupled intrinsic Josephson junctions in which the longitudinal Josephson plasma, i.e., longitudinal collective phase oscillation modes, is excited. The scheme of energy-level quantization in the collective oscillatory states is clarified in the N-junction system. When the MQT occurs from the single-plasmon states excited by microwave irradiation in the multi-photon process to the uniform voltage state, our theory predicts that the escape rate is proportional to N^2. This result is consistent with the recent observation in Bi-2212 intrinsic Josephson junctions.

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Since the discovery of macroscopic quantum tunneling (MQT) in YBCO grain boundary junctions [1] and Bi-2212 intrinsic Josephson junctions (IJJs) [2], a renewed interest has been aroused on the MQT in the Josephson effect from viewpoints of both basic physics and quantum-devise applications. In Bi-2212 IJJs the observed crossover temperature at which MQT appears is about 1K[2], which is 1-order higher than that in conventional single-junction systems [3, 4], and, furthermore, the dissipation effect that prevents a quantum tunneling is very weak in spite that the order parameter of the high-T_c cuprates has the d-wave symmetry with nodes. This is because the Josephson plasma frequency in IJJs is far higher compared with artificially-made S/I/S Josephson junctions [5] and the transfer-integral along the c-direction vanishes for the nodal quasi-particles. These remarkable features in IJJs give great promise for quantum-devise applications. In addition, MQT in IJJs is expected to provide a new physics originating from the atomic-scale multi-junction structure.

Recently, Jin et al. observed MQT in the switching events to the uniform voltage state in Bi-2212 IJJs where all the junctions are switched into the voltage state[6]. The switching rate observed in the multi-photon process is greatly enhanced, depending on the number of stacked junctions, N. The observed enhancement in the collective switching is proportional to N^2, which suggests that collective motion of the phase differences in IJJs is responsible for the quantum tunneling. In IJJs the longitudinal Josephson plasma is known to exist as the collective motion of the phase differences that propagates in the stacking direction of the junctions [7]. In this paper we formulate a theory for MQT of the collective longitudinal plasma modes. The scheme of quantum energy levels for the collective motion of the phase differences in capacitively-coupled N intrinsic junctions is clarified. Our theory predicts that the switching rate in the multi-photon process is proportional to N^2.

Consider a stack of intrinsic Josephson junctions having tiny in-plane area of $W \sim 1\mu m^2$. In the absence of an external magnetic field the phase of the order parameter φ_ℓ, ℓ being the layer index, can be considered uniform along the in-plane direction. In this case the coupling between junctions in equilibrium state is brought about by the long-range Coulomb interaction between charges induced in the superconducting layers with an atomic-scale layer thickness as discussed in [7], that is, the coupling originates from the incomplete charge screening between junctions. The charge and phase dynamics in this case is well described by the Lagrangian [8],

$$\mathcal{L} = W \sum_{\ell=1}^{\infty} \Big\{ \frac{s}{8\pi\mu^2}\big(A_\ell^0 + \frac{\hbar}{e^*}\dot{\varphi}_\ell\big)^2 + \frac{\epsilon d}{8\pi} E_{\ell,\ell-1}^2 - E_J[1-\cos(\varphi_\ell - \varphi_{\ell-1})] \Big\}, \qquad (1)$$

where A_ℓ^0 is the scalar potential at the ℓth superconducting layer, $E_{\ell,\ell-1}$ is the electric field inside the insulating layer with the dielectric constant ϵ between ℓth and $(\ell-1)$th superconducting layers, and s and d are, respectively, the thicknesses of the superconducting and insulating layers. The Josephson coupling energy in Eq.(1) is defined as $E_J = \hbar j_c/|e^*|$, j_c being the Josephson critical current density. The first term including A_ℓ^0 in Eq.(1) corresponds to the charging energy which leads to the finite charge compressibility [9]. To see this we note that the canonical momentum which is conjugate to φ_ℓ is given as

$$p_\ell = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_\ell} = \frac{\hbar}{e^*}\cdot\frac{s}{4\pi\mu^2}\big(A_\ell^0 + \frac{\hbar}{e^*}\dot{\varphi}_\ell\big) = \hbar s n_\ell. \qquad (2)$$

where n_ℓ is understood to be the density fluctuation of the Cooper-pairs in the ℓth layer. Then, the first term in Eq.(1) increases the free energy by the amount of $\propto n_\ell^2$, which leads to the finite charge compressibility discussed by van der Marel and Tsvetkov [9]. The effect of the charge compressibility has been observed in optical properties of high-T_c superconductors [10, 11].

The Lagrangian (1) yields the Hamiltonian of the form[12],

$$\mathcal{H} = \frac{1}{2}\sum_{\ell m} C^{-1}_{\ell m} Q_\ell Q_m + E_J \sum_{\ell}\big[1-\cos(\varphi_\ell - \varphi_{\ell-1})\big], \quad (3)$$

where Q_ℓ is the total charge of the ℓth layer, i.e., $Q_\ell = e^* W s n_\ell$, and $C^{-1}_{\ell m}$ is the inverse capacitance matrix. Note that the condition $d^2 \ll W$ is fulfilled even in the intrinsic Josephson junctions with a small in-plane area of $W \sim 1\mu\mathrm{m}^2$, since $d \simeq 1.2$nm. From this fact one understands that the electric field generated in the junctions is well confined inside the stack of the junctions and its direction is perpendicular to the junctions, and as a result, the 1D Coulomb potential is realized almost completely. Then, the inverse mutual capacitance diverges linearly as[9]

$$C^{-1}_{\ell m} \to \frac{4\pi d}{\varepsilon W}\ell, \qquad \text{for } \ell \gg m. \quad (4)$$

This result indicates that the long-range nature of the capacitive coupling between junctions should be correctly incorporated, that is, any truncation of the inverse capacitance matrix in Eq.(3) cannot be accepted in the intrinsic Josephson junctions. As shown in [12], the Hamiltonian (3) can be transformed into more tractable form by the canonical transformation. We choose (θ_ℓ, u_ℓ) defined as

$$\theta_\ell = \varphi_\ell - \varphi_{\ell-1}, \qquad u_\ell = \sum_{m=\ell}^{\infty} p_m, \quad (5)$$

as the canonical variables instead of (φ_ℓ, p_ℓ). It is easy to see that the canonical commutation relation $[\theta_\ell, u_m] = i\hbar\delta_{\ell m}$ holds if $[\varphi_\ell, p_m] = i\hbar\delta_{\ell m}$. The Hamiltonian (3) is greatly simplified if one uses the canonical variables (θ_ℓ, u_ℓ) as [12]

$$\mathcal{H} = \sum_{\ell=1}^{\infty}\Big\{\frac{E_c}{\hbar^2}\Big[(1+2\alpha)u_\ell^2 - 2\alpha u_\ell u_{\ell+1}\Big] + E_J\big[1-\cos\theta_\ell\big]\Big\}, \quad (6)$$

with $E_c = 2\pi d e^{*2}/W\epsilon$ and $\alpha = \epsilon\mu^2/sd$. Note that the Josephson coupling term is diagonal and the coupling between junctions survives only for nearest neighbors. The coupling constant α is identical with the one introduced in the capacitively-coupled classical intrinsic Josephson junctions[7, 8]. In Bi-2212 α takes a value of $\sim 0.1-0.2$, depending on the doping level[8, 13, 14]. Let us discuss the small quantum phase oscillations in the N-junction system. We consider the case of $N \gg 1$ and neglect the boundary effect in the following calculations. For small phase oscillations one can use the approximation, $1-\cos\theta_\ell \simeq \frac{1}{2}\theta_\ell^2$. Using the periodic boundary condition, we express u_ℓ and θ_ℓ in terms of the Fourier series expansions as $u_\ell = N^{-1/2}\sum_n \hat{u}_n \exp(i2\pi n\ell/N)$, $\theta_\ell = N^{-1/2}\sum_n \hat{\theta}_n \exp(i2\pi n\ell/N)$, where the Fourier components $(\hat{u}_n, \hat{\theta}_n)$ satisfy the commutation relation, $[\hat{\theta}_n, \hat{u}_{-m}] = i\hbar\delta_{\ell m}$. The Hamilotonian (6) in the harmonic approximation can be expressed in terms of the Fourier components as

$$\mathcal{H} = \sum_{n=-n_c}^{n_c}\Big\{\frac{E_c}{\hbar^2}\big[1+2\alpha(1-\cos\frac{2\pi n}{N})\big]\hat{u}_n\hat{u}_{-n} + \frac{1}{2}E_J\hat{\theta}_n\hat{\theta}_{-n}\Big\}, \quad (7)$$

where n_c is given as $N = 2n_c+1$. Then, in terms of the creation and annihilation operators c_n and $c_n^\dagger$ defined as $\hat{\theta}_n = (c_n + c^\dagger_{-n})/(\sqrt{2}K_n)$ and $\hat{u}_n = -i\hbar K_n(c_n - c^\dagger_{-n})/\sqrt{2}$ with $K_n = (E_J/\omega_n)^{1/2}$, one can diagonalize the Hamitonian as

$$\mathcal{H} = \hbar\sum_{n=-n_c}^{n_c}\omega_n(c_n^\dagger c_n + \frac{1}{2}), \quad (8)$$

where

$$\omega_n = \omega_{\rm pl}\sqrt{1+2\alpha(1-\cos\frac{2\pi n}{N})}, \quad (9)$$

with $\omega_{\rm pl}$ being the Josephson-plasma frequency defined as $\hbar\omega_{\rm pl} = \sqrt{2E_cE_J}$. This phase oscillation mode is identified with the quantum version of the longitudinal Josephson plasma. The dispersion relation given in Eq.(9) is the same as the classical one[7, 8]. The longitudinal Josephson plasma has been observed in the microwave absorption experiments in Bi-2212[15, 16]. Its frequency $\omega_{\rm pl}$ is located in the range of a few hundreds GHz at $T = 0$K[17]. As seen from Eq.(9), we have N plasma modes in the N-junction system, which is crucially different from the single-junction system, and their frequencies range from $\omega_{n=0} = \omega_{\rm pl}$ up to $\omega_{n=\pm N/2} = \sqrt{1+4\alpha}\,\omega_{\rm pl}$. In the case of $\alpha \sim 0.1$ (under-doped Bi-2212 case) the relation, $\sqrt{1+4\alpha}\,\omega_{\rm pl} - \omega_{\rm pl} \ll \omega_{\rm pl}$, holds. Then, the lowest single-plasmon state is well above the highest zero-plasmon state (see Fig.1). From this observation one expects the energy scheme as depicted schematically in Fig.1 for the capacitively-coupled Bi-2212 intrinsic Josephson junctions in the quantum regime, if the harmonic approximation for the phase oscillation modes is assumed to be valid up to the single-plasmon state. In this figure we also describe the washboard-like effective potential which confines the collective phase oscillation modes. The tilt of the potential can be generated by a bias current. Note that in the presence of a bias current I the plasma frequency is shifted as $\omega_{\rm pl} \to \omega_{\rm pl}(\tilde{I}) = \omega_{\rm pl}(1-\tilde{I}^2)^{1/4}$ with $\tilde{I} = I/j_c$ by the effect of the coupling term $-E_J\tilde{I}\sum_\ell \theta_\ell$. If $f_{\rm pl} = \omega_{\rm pl}/2\pi \sim 100$GHz, we have the level spacings $\hbar\omega_{\rm pl} \sim 4.8$K between zero-plasmon and single-plasmon states and $\hbar\omega_{n+1} - \hbar\omega_n \sim 0.23$K between adjacent plasma modes for $\alpha = 0.1$ and $N = 50$. From this estimation the quantum effect originating from the discreteness of the n-plasmon states may be expected at $T \sim 1$K. The level splitting between the plasma modes can be seen at temperatures below ~ 100mK.

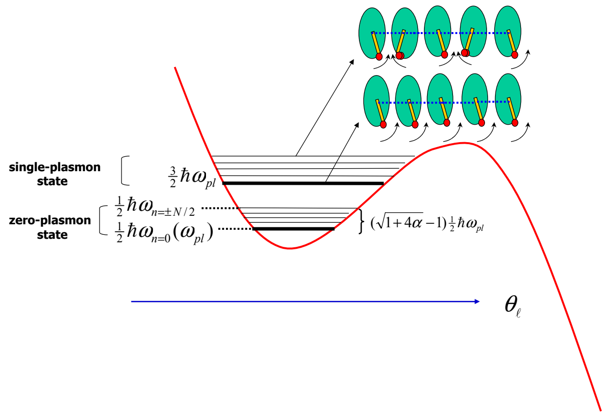

FIG. 1: Energy-level quantization of the longitudinal collective phase oscillation modes in the presence of a bias current. In the N-junction system single-plasmon states are composed of N oscillation modes which are nearly degenerate in the case of $\alpha \ll 1$. The zero-plasmon state is formed by the zero-point motion of the N plasma modes.

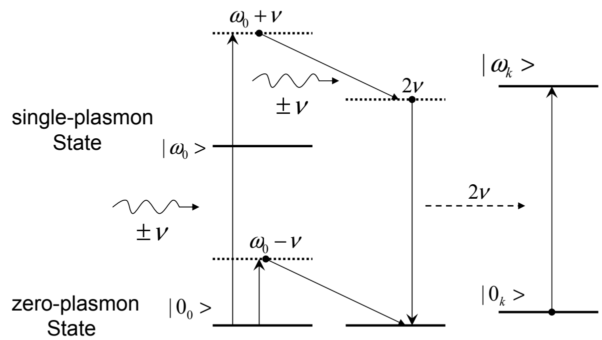

FIG. 2: Two-photon process that causes the transition $|0> \rightarrow |\omega_k>$ in the third-order perturbation. In this process the virtual plasmon of mode $k=0$ with energy 2ν, which is excited in the two-photon process, excites a plasmon of mode k by the interaction V_3^k when $2\nu = \omega_k$. In this figure $|0_0>$ and $|0_k>$ denote, respectively, the vacuum states of 0th and kth plasma oscillations, and $|\omega_0>$ and $|\omega_k>$ the single-plasmon states of these oscillations.

Let us now study the MQT in the intrinsic Josephson junctions in which the longitudinal Josephson plasma is excited. Suppose that the single-plasmon states show the MQT to the voltage state. Since the longitudinal plasma oscillations are the coherent motion of all the junctions, it is reasonable to assume that the voltage state switched from the single-plasmon state is uniform, that is, all the junctions are in the voltage state. This switching phenomenon should be discriminated from the MQT to the first resistive branch[2]. The quantum switching to the first resistive branch is induced by the MQT of the nonlinear localized mode such as the discrete breather[18] to the localized rotating mode[8, 19]. The MQT of the discrete breather will be discussed in a forthcoming paper.

In this paper we study the photon-assisted collective MQT in the intrinsic Josephson junctions, because the MQT to the uniform resistive state has been observed only in the systems under the microwave irradiation up to now[6]. In the following we focus on the origin of the intriguing N^2-dependence of the MQT rate recently observed in Bi-2212. For this we consider the following third-order anharmonic interaction term that appears in the presence of a bias current,

$$V_3 = -\frac{\gamma}{3!}E_J\tilde{I}\sum_\ell \theta_\ell^3$$

$$= -\frac{\gamma E_J\tilde{I}}{3!\sqrt{N}}\sum_{n_1+n_2+n_3=0,\pm N}\hat{\theta}_{n_1}\hat{\theta}_{n_2}\hat{\theta}_{n_3}, \qquad (10)$$

with γ being some constant. Note that this interaction term induces the coupling among the plasma modes, i.e., the mode-mode coupling. The conservation law in Eq.(10), $n_1+n_2+n_3=0,\pm N$, comes from the periodic boundary condition. In the real systems composed of $N \sim 10-100$ junctions this conservation law is understood to hold approximately. One expects that this anharmonic interaction term becomes important for the excited states, i.e., the single-plasmon states, as the bias current is increased.

Let us investigate the excitations of the plasma mode by microwave irradiation, that is, the transition from the ground state $|0>$ to kth single-plasmon state $|\omega_k>$ (see Fig.2). We assume that the coupling between microwave and the phase differences is generated by the oscillating current induced by the microwave. Then, the interaction with the microwave of frequency ν is described by the term as

$$V_M(t) = E_J\tilde{I}_M\sin\nu t\sum_\ell\theta_\ell = \sqrt{N}g_M(c_0+c_0^\dagger)\sin\nu t, \qquad (11)$$

where $g_M = E_J\tilde{I}_M/(2\sqrt{2}K_0)$, $\tilde{I}_M$ being the intensity of the normalized oscillating current. From Eq.(11) one understands that the microwave can excite only the uniform plasma mode of $n=0$, $|\omega_0>$, in the one-photon process. The transition rate $1/\tau^{(1)}$ in this process is easily derived as

$$\frac{1}{\tau^{(1)}} = N\pi g_M^2\delta(\omega_{\rm pl}(\tilde{I})-\nu). \qquad (12)$$

On the other hand, in the multi-photon processes in which the mode-mode coupling is incorporated, the plasma modes with $k \neq 0$ can be excited as shown in the following. Note that Eq.(10) includes the interaction term,

$$V_3^k = -\frac{f_k}{\sqrt{N}}c_k^\dagger c_{m_k}c_{m_k}^\dagger, \qquad (13)$$

where $f_k = \gamma E_J \tilde{I}(8K_k K_{m_k}^2)^{-1/2}$ and m_k is the index satisfying the relation $k + 2m_k = 0, \pm N$. Let us study the third-order perturbative processes caused by the interaction $\mathcal{H}_{\rm int} = V_M(t) + V_3^k + V_3^{k\dagger}$. The two-photon process is included in the third-order perturbation and the transition, $|0> \rightarrow |\omega_k>$, is possible in the presence of V_3^k as seen in Fig.2. Note that $V_M(T) \propto \sqrt{N}$ and $V_3^k \propto (\sqrt{N})^{-1}$. Hence, one understands that the transition matrix element $< f|i > = < \omega_k|U^{(3)}(t,-\infty)|0 >$ in the two-photon process is proportional to $\sqrt{N}$, where $U^{(3)}(t,-\infty)$ is the third-order time-evolution operator given by

$$U^{(3)}(t,-\infty) = (-\frac{i}{\hbar})^3 \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t_1} \mathrm{d}t_2 \int_{-\infty}^{t_2} \mathrm{d}t_3 \; \mathrm{e}^{\epsilon(t_1+t_2+t_3)} \mathcal{H}_{\rm int}(t_1)\mathcal{H}_{\rm int}(t_2)\mathcal{H}_{\rm int}(t_3)\Big|_{\epsilon\to 0}, \quad (14)$$

and thus the transition rate, $\tau_k^{-1} = \frac{\mathrm{d}}{\mathrm{d}t}| < \omega_k|U^{(3)}(t,-\infty)|0 > |^2$, is proportional to N. In fact, from the simple calculations we obtain

$$\frac{1}{\tau_k} = N \frac{\pi(g_M^2 f_k)^2}{(\omega_{\rm pl}(\tilde{I}) - \nu)(\omega_k + \omega_{\rm pl}(\tilde{I}) - \nu)} \delta(\omega_k - 2\nu). \quad (15)$$

Then, if the MQT rate of the single-plasmon state $|\omega_k>$ to the voltage state is given by Γ_k, the photon-assisted switching rate is obtained as $\tau_k^{-1}\Gamma_k$. In the MQT at temperatures around $\sim$ 1K the splitting between the plasma modes is smeared out for the systems with $N \sim 50$. Then, we find the total switching rate in the two-photon process for $\nu \sim \omega_{pl}/2$ as

$$\Gamma^{(2)} = \sum_{k=-n_c}^{n_c} \frac{1}{\tau_k}\Gamma_k \propto N^2 \frac{\pi(g_M^2 f_0)^2}{\omega_{\rm pl}(\tilde{I})^2}\Gamma_0, \quad (16)$$

which is proportional to N^2. The N^2-dependence of the switching rate is also obtained in the three-photon process. Note that the transition, $|0> \rightarrow |\omega_k>$, in the three-photon process is possible in the fifth-order perturbative process with respect to $\mathcal{H}_{\rm int}$. Then, from Eqs.(10) and (11) we expect the relation, $< f|i > \propto (\sqrt{N})^3 \times (\sqrt{N})^{-2} = \sqrt{N}$, for the three-photon process in the fifth-order perturbation, which is the same as in the two-photon process. This observation indicates that the total switching rate is also proportional to N^2 in the three-photon process. Thus, one may conclude that the MQT rate to the uniform resistive state is proportional to N^2, if the MQT happens collectively in the multi-photon process. This result is consistent with the recent experimental result in Bi-2212 intrinsic Josephosn junctions[6]. Note that the N^2-dependence dose not appear in the switching rate by way of the single-photon process. This is because the single-plasmon state of $k \neq 0$ cannot be available in the single-photon process. Thus, our theory predicts that the switching rate in the single-photon process is proportional to N. Furthermore, from the result given in Eqs.(15) and (16) it is also predicted that the frequency spectrum of the switching rate, i.e., $\Gamma^{(2)}(\nu)$, splits into $N/2$ peaks having centers at $\nu = \omega_k/n$ at low temperatures, say $T \sim 10$mK, in the n-photon process if the damping originating from the coupling with the quasiparticles or environment is weak enough.

In summary we have formulated MQT for the collective longitudinal Josephson plasma modes in intrinsic Josephson junctions, assuming that the switching events to the uniform voltage state occur from the single-plasmon states. The scheme of energy-level quantization for the collective oscillation modes has been clarified in the case of Bi-2212 IJJs. In the N-junction system the single-plasmon states are composed of N different oscillation modes whose frequencies range from $\omega_{\rm pl}$ to $\sqrt{1+4\alpha}\omega_{\rm pl}$. These modes can be excited by the microwave irradiation in the multi-photon process. Then, there are N channels in the MQT of the collective phase oscillation modes in the multi-photon process. We have also shown that the transition rate to each single-plasmon state in terms of microwave irradiation is proportional to N. Therefore, the photon-assisted switching rate to the uniform voltage state is proportional to N^2 in the multi-photon process around 1K.

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